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GRAPHS FOR THE DETERMINATION OF THE TEMPERATURE FIELD IN TIME IN RODS OR SHEETS OF FINITE LENGTH WITH INTERNAL HEAT LIBERATION

A. Kessler

Translation of "Grafiki dlya Opredeleniya Temperaturnogo Polya vo Vremeni v Sterzhnyakh ili Plastinakh Konechnoy Dliny s Vnutrennim Teploobrazovaniyem"
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Symbols

k	temperature conductance;
λ	coefficient of heat conductance;
z	specific heat liberation;
ρc	heat capacity per unit volume;
l	length of rod;
Z	total heat quantity liberated in the rod per unit time.

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ABSTRACT. The problem of heating in time in a rod heat insulated on the surface with internal heat liberation and having a constant temperature at one end, is solved. The solution of this problem depends only on the relative position of $\chi = x/l$ and on the Fourier criterion $Fo = kt/l^2 = \lambda t/\rho c l^2$. The equations necessary for solving this problem are given.

In one of our earlier works [1] we solved the problem of heating in time in sheets of finite thickness and in rods of finite length with internal heat liberation and cooled on the surface for all four types of boundary conditions [2]. In the case of a rod (laminar), heat insulated on the surface with internal heat liberation, with a constant temperature /134[1]

$$T(x,\ 0) = 0,\ \left(\frac{\partial T}{\partial x}\right)_{x=0} = 0$$

maintained on one end of the rod, where $T(l,t) = 0$, the solution of the problem depends only on the relative position of $\chi = x/l$ and on the Fourier criterion $Fo = kt/l^2 = \lambda t/\rho c l^2$. If $\theta(\chi, Fo)$ denotes heating related to a stationary temperature field $\lim T(\chi, Fo)$, we then have

$$T(\chi,\ Fo) = \theta(\chi,\ Fo) \lim_{Fo \to \infty} T(\chi,\ Fo),$$

for the instantaneous temperature distribution, where

$$\theta(\chi, Fo) = 1 - \frac{4}{1-\chi^2} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\delta_n^3} \cos \delta_n \chi \exp[-Fo\, \delta_n^2],$$

$$\delta = \frac{(2n-1)}{2}\pi \quad \text{and} \quad \lim_{Fo \to \infty} T(\chi,\ Fo) = \frac{zl^2}{2\lambda}(1-\chi^2).$$

[1] Numbers in the margin indicate pagination in the foreign text.

After finding the values of $\theta(\chi, \mathrm{Fo})$ in Figure 1, it is easy to determine $T(\chi, \mathrm{Fo})$ or $T(x,t)$ for all parameters.

To determine the quantity of heat liberated by the rod at its end $x = l$ per unit time in cross section P

$$q(\mathrm{Fo}) = -\lambda P \left\{\frac{dT}{dx}\right\}_{x=l}$$

we find

$$q(\mathrm{Fo}) = Z\varkappa(\mathrm{Fo}), \quad \varkappa(\mathrm{Fo}) = 1 - 2\sum_{n=1}^{\infty} \frac{1}{\delta_n^2} \exp[-\mathrm{Fo}\,\delta_n^2].$$

The values of $x(\mathrm{Fo})$ should be taken from Figure 2.

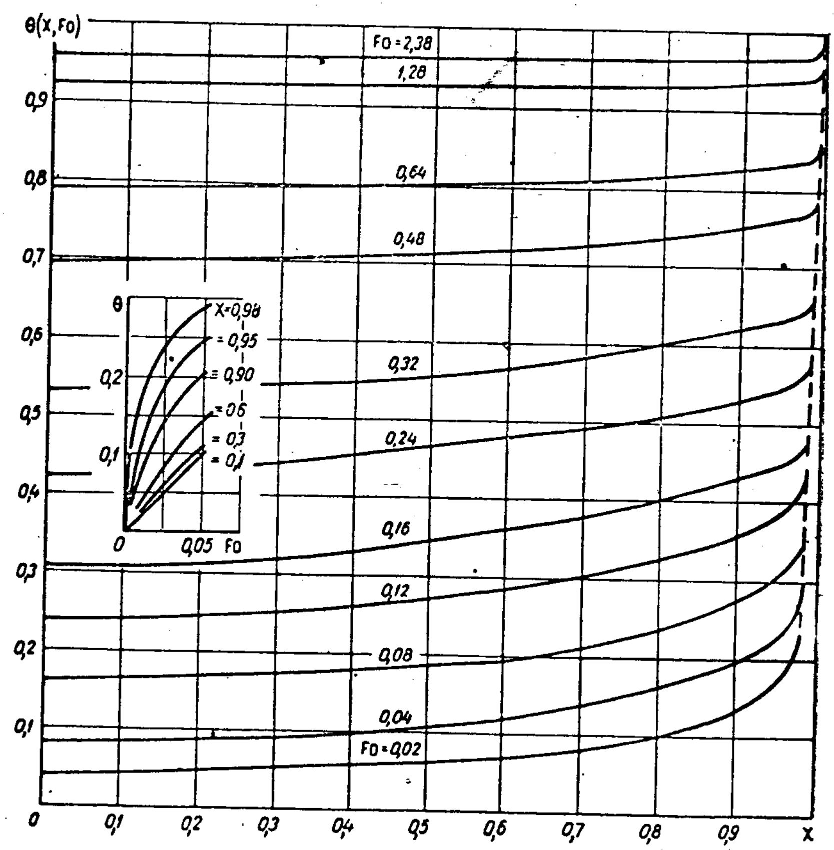

Figure 1 /135

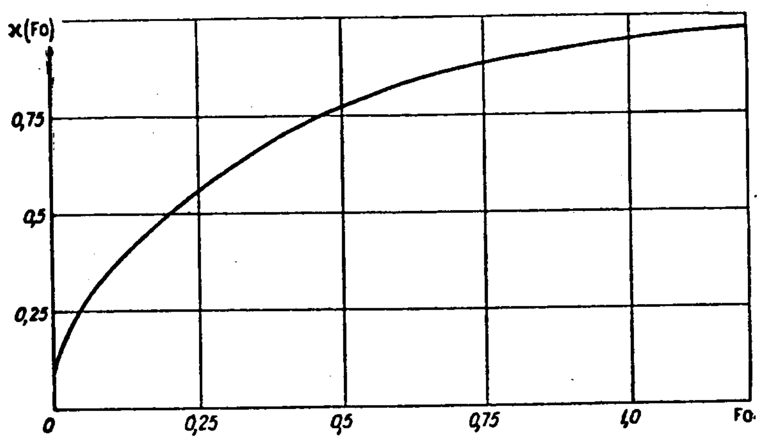

Figure 2

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1. Kessler, A., *Aplikace Matematiky*, No. 3, p. 190, 1958, Prague. /136
2. Lykov, A. V., *Teoriya Teploprovodnosti* [Theory of Heat Conductance], GITTL Press, 1952.

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